Analysis of Full Wave 2D to 3D Propagation Models and Ray Tracing for Indoor Environments

Ian Kavanagh, Conor Brennan
School of Electronic Engineering, Dublin City University, Ireland

***Abstract*—In indoor propagation modelling a greater level of accuracy is being sought after to design energy efficient radio access nodes and determine the optimum locations for base stations to keep up with wireless communications demand. The volume electric field integral equation (VEFIE) is a full wave method derived from Maxwell's equations and capable of providing the desired level of accuracy. The 3D formulation is slow, whilst the 2D version is quick but lacks accuracy. This has led to the development of 2D to 3D models designed to provide quick accurate 3D predictions. The 2D to 3D models are compared against the VEFIE and shown to be more accurate than the 2D model and quicker than the 3D variant. They are also compared against ray tracing, the current most popular method for indoor propagation modelling, and shown to be more favourable.**

I. Introduction

The explosion, in recent times, of the use of mobile devices has put a huge strain on wireless communications infrastructure. The increased demand has led to new developments in energy efficient indoor mobile communications systems [1]. Correspondingly, there is a growing demand for accurate propagation models to serve as design tools and in optimising the location and positions of these base stations [2]. These models should include as much of the geometry of the environment as possible but yield a solution within a reasonable time frame. This is extremely challenging given the special characteristics of indoor environments such as the variety of building materials and layouts encountered.

Current trends for indoor propagation modelling use empirical or approximate models that lack the desired accuracy but can provide rapid predictions. The COST 231 Multi Wall model [3] and the Adjusted Motley-Keenan model [4] are two very popular empirical models. They are very quick and simple to apply but are limited in their accuracy, particularly in complex indoor environments. Instead, significant research is focused on ray tracing. Ray tracing a fast method that originates from the approximation of Maxwell's equations at high frequencies. It has been augmented with the Geometrical Theory of Diffraction (GTD) and Uniform Theory of Diffraction (UTD) in [5], [6] respectively to include the diffraction phenomenon and enhance its accuracy. More recently, the inclusion of diffuse scattering in ray tracing has seen significant interest [7] to further enhance its accuracy. This trend towards increasing the accuracy of ray tracing and consequently its computation time suggests it may be beneficial to take a different approach - to start with an accurate full wave method and reduce its computational burden.

The volume electric field integral equation (VEFIE) is a full wave method derived from Maxwell's equations. Its solution provides accurate information about the propagation of electromagnetic waves [8]. We have previously investigated using the VEFIE for indoor propagation modelling [9]. The 3D formulation of the VEFIE is very accurate but takes a long time to reach a solution. The 2D formulation on the other hand is very quick but lacks accuracy due to its 2D nature. In this paper, we examine two methods for correcting the 2D formulation and computing accurate 3D predictions very quickly. The first method is heuristic and corrects the 2D VEFIE primarily based on the difference in the incident fields, we assume a dipole is used as the antenna. The second method is more general and involves the solution of two modified 2D simulations. The correction methods, called 2D to 3D models, are compared against the 2D and 3D formulations of the VEFIE specifically looking at their accuracy and efficiency. Comparisons are also made with ray tracing, the currently most popular method for indoor propagation modelling.

This paper is organised as follows. The 3D and 2D formulations of the VEFIE are presented in Section II. Section III outlines the 2D to 3D models and how their solution is found. Numerical results analysing the accuracy and efficiency of the 2D to 3D models is presented in Section IV. Whilst, Section V concludes the paper with a brief summary and some concluding remarks.

II. Formulations

The volume electric field integral equation (VEFIE) is derived from Maxwell's equations using the volumetric equivalence principle [8]. It is well suited to the problem of indoor propagation modelling [9] due to the ease with which inhomogeneous scatterers can be modelled and the lack of a need for absorbing boundary conditions. It expresses the total electric field at a point in terms of an incident field and a scattered field,

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^i(\mathbf{r}) + \mathbf{E}^s(\mathbf{r}) \tag{1}$$

where $\mathbf{E}^i(\mathbf{r})$ is the incident field and

$$\mathbf{E}^s(\mathbf{r}) = k_0^2 \left(1 + \frac{1}{k_0^2}\nabla\nabla\cdot\right) \int_V G(\mathbf{r},\mathbf{r}')\chi(\mathbf{r}')\mathbf{E}(\mathbf{r}')d\mathbf{r}' \tag{2}$$

is the scattered field. $G(\mathbf{r},\mathbf{r}')$ is the 3D scalar free space Green's function given by

$$G(\mathbf{r},\mathbf{r}') = \frac{e^{-\jmath k_0|\mathbf{r}'-\mathbf{r}|}}{4\pi|\mathbf{r}'-\mathbf{r}|} \tag{3}$$

and

$$\chi(\mathbf{r}') = \frac{k^2(\mathbf{r}')}{k_0^2} - 1 \tag{4}$$

is a contrast function representing the geometry of the problem. $\mathbf{r}$ is the point we are computing the field at and $\mathbf{r}'$ represents all of the points in the volume V that are being integrated over. k_0 is the free space wave number and $k^2 = \omega^2\mu\epsilon - \jmath\omega\mu\sigma$.

In order to solve the VEFIE we first discretise it, which results in a matrix equation that can be solved with an iterative solver. We use a weak-form discretisation process [10] to discretise the 3D formulation shown in (1) and (2). This results in a matrix equation of the form

$$\mathbf{V} = \mathbf{Z}\mathbf{E} = (\mathbf{I} - k_0^2\mathbf{G}\mathbf{D} - \mathbf{H}\mathbf{G}\mathbf{D})\mathbf{E} \tag{5}$$

where $\mathbf{E}$ and $\mathbf{V}$ are column vectors representing the unknown total electric field and the known incident electric field,

$$\mathbf{E} = \begin{bmatrix}\mathbf{E}_x\\ \mathbf{E}_y\\ \mathbf{E}_z\end{bmatrix}, \mathbf{V} = \begin{bmatrix}\mathbf{E}_x^{\mathrm{i}}\\ \mathbf{E}_y^{\mathrm{i}}\\ \mathbf{E}_z^{\mathrm{i}}\end{bmatrix} \tag{6}$$

$\mathbf{I}$ is the identity matrix, $\mathbf{G}$ represents the Green's function in (3) and has the form

$$\mathbf{G} = \begin{bmatrix}\mathbf{G_T} & 0 & 0\\ 0 & \mathbf{G_T} & 0\\ 0 & 0 & \mathbf{G_T}\end{bmatrix} \tag{7}$$

where each $\mathbf{G_T}$ has block Toeplitz symmetry and $\mathbf{D}$ is a diagonal contrast matrix corresponding to the values of (4). $\mathbf{H}$ is a sparse matrix containing a suitable numerical approximation to the $\nabla\nabla\cdot$ operation. We solve (5) using an iterative solver accelerated by use of the fast Fourier transform (FFT).

A. 2D Formulation

The 2D formulation of the VEFIE is derived from the 3D equation by considering the case of transverse magnetic polarisation (TM^z) and scatterers which extend to infinity in the z direction. This leads to

$$E_z(\mathbf{r}) = E_z^i(\mathbf{r}) + k_0^2 \int_S G(\mathbf{r},\mathbf{r}')\chi(\mathbf{r}')E_z(\mathbf{r}')d\mathbf{r}' \tag{8}$$

where this time $G(\mathbf{r},\mathbf{r}')$ is the 2D scalar Green's function given by

$$G(\mathbf{r},\mathbf{r}') = \frac{1}{4\jmath}H_0^{(2)}(k_0|\mathbf{r}'-\mathbf{r}|) \tag{9}$$

and $H_0^{(2)}(k_0|\mathbf{r}'-\mathbf{r}|)$ is the zeroth order Hankel function of the second kind. In order to solve this equation we, again, discretise it, which results in a matrix equation of the form

$$\mathbf{V} = \mathbf{Z}\mathbf{E} = (\mathbf{I} - k_0^2\mathbf{G}\mathbf{D})\mathbf{E} \tag{10}$$

and solve it using an iterative solver accelerated with the FFT.

B. Incident Field Models

In this paper for the purpose of analysing and simulating the propagation of electromagnetic waves indoors we consider the incident field as a vertical dipole. This is represented by

$$\mathbf{E}^i(\mathbf{r}) \cong \hat{\theta}\jmath\omega\mu_0\frac{Il}{4\pi R}e^{-\jmath k_0 R}\sin\theta \tag{11}$$

in the far field where I is the amount of current flowing in the dipole and l is its length. $R = |\mathbf{r} - \mathbf{r}_a|$, where $\mathbf{r}$ is the point of interest and $\mathbf{r}_a$ is the location of the antenna. The near field radiation characteristics of the dipole can be ignored as we ensure $k_0 R \gg 1$. In 2D the incident field reduces in a similar fashion to the VEFIE and is represented by a line source given by

$$E_z^i(\mathbf{r}) = \frac{1}{4\jmath}H_0^{(2)}(k_0 R) \tag{12}$$

III. 2D TO 3D MODELS

As we shall see in Section IV the solution of the 3D VEFIE is quite slow, whereas that of the 2D VEFIE is very fast but lacks accuracy. This is for two reasons. Firstly, in 3D we must model 3 field components, x, y and z, as opposed to one, z, in 2D. Secondly, the number of unknown values in the 3D problem grows as $N_xN_yN_z$, where N_x, N_y and N_z are the number of discretisations in the x, y and z directions, rather than N_xN_y in the 2D case. Thus, a more computationally efficient approach to propagation modelling is to perform 2D simulations and somehow correct the fields to account for 3D propagation. This section presents two 2D to 3D models based on 2D simulations that produce accurate 3D results.

A. Heuristic Model

The first 2D to 3D model, the heuristic model, assumes the dominant propagation mechanism is the incident field and corrects the 2D fields based on this and experimentally determined values. The correction is based on the difference in the 2D and 3D formulations for the VEFIE and the incident fields. These differences are detailed in [11]. Due to the nature of the correction a slight distinction is made between points in line of sight (LOS) and points in non line of sight (NLOS) regions.

In areas that have LOS with the antenna the dominant component of the electric field is due to the incident field. The scattered field strength is relatively weak in typical LOS scenarios found in indoor environments. A suitable accuracy can be achieved by correcting the solution of the 2D VEFIE based primarily on the incident field. Thus, our LOS correction is given as

$$E_{3D}(\mathbf{r}) = \beta\frac{E_{2D}(\mathbf{r})}{\sqrt{R}}\sin\theta \tag{13}$$

where $E_{3D}(\mathbf{r})$ is the total field in three dimensions and $E_{2D}(\mathbf{r})$ is the solution of the 2D VEFIE. R is the same as that used for the incident fields in Section II-B and $\mathbf{r}$ is the point being corrected. β is given by

$$\beta = \omega\mu_0\frac{Il}{4\sqrt{2\pi}}\sqrt{k_0} \tag{14}$$

where I and l are the same values used in (11). θ is given by

$$\theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{\sqrt{z}}\right) \tag{15}$$

In NLOS regions, those that are shadowed from the antenna, the scattered field plays a more important role but it is still dependant on the incident field. This means fields in NLOS areas can be corrected with a small variation on (13) by

$$E_{3D}(\mathbf{r}) = \alpha + \beta\frac{E_{2D}(\mathbf{r})}{\sqrt{R}} \tag{16}$$

where

$$\alpha = \begin{cases} 0 & z \le 2\lambda_0 \\ \frac{z}{100\lambda_0} & z > 2\lambda_0 \end{cases} \tag{17}$$

B. Hybrid Model

The hybrid model represents an improvement over the heuristic model as it is more general and not as dependant on the incident field being a dipole. The solution of the hybrid model is based on the average of two modified 2D simulations. As shown above, the 2D VEFIE can be expressed in matrix form as $\mathbf{V} = (\mathbf{I} - k_0^2\mathbf{G}\mathbf{D})\mathbf{E}$. This is the basis of the hybrid model.

The final solution of the hybrid model, $\mathbf{E}$, is given by

$$\mathbf{E} = \frac{1}{2}(\mathbf{E}_2 + \mathbf{E}_3) \tag{18}$$

$\mathbf{E}_2$ is the solution of

$$\mathbf{V}_{3D} = (\mathbf{I} - k_0^2\mathbf{G}_{2D}\mathbf{D})\mathbf{E}_2 \tag{19}$$

where $\mathbf{V}_{3D}$ is the 3D incident field for the plane through the z axis of interest. In this study, we use a dipole and focus on the plane cutting through the centre of its axis. $\mathbf{G}_{2D}$ represents the 2D Green's function given by (12). $\mathbf{E}_3$ is the solution of

$$\mathbf{V}_{3D} = (\mathbf{I} - k_0^2\mathbf{G}_{3D}\mathbf{D})\mathbf{E}_3 \tag{20}$$

where $\mathbf{G}_{3D}$ represents the 3D Green's function given by (3) with z equal to 0. The effect of using the 3D Green's function in the 2D VEFIE computes the integral in the scattered field over an infinitesimally small plane as opposed to using the 2D Green's function which computes the integral from $-\infty$ to $+\infty$. The use of the 3D Green's function causes an underestimation of the strength of the scattered field, whilst the 2D Green's function causes an overestimation of it. The resultant average produces a model that agrees very well with the full 3D VEFIE.

IV. Numerical Results

In this section numerical results are presented demonstrating the accuracy of the 2D to 3D models and improved efficiency over the full 3D VEFIE for the building shown in Figure 1. The models are also compared with ray tracing. The ray tracing model computes up to third order reflections and transmissions but does not include diffraction or diffuse scattering. All of the simulations were performed on a laptop with a 2.6GHz Core $i7$ CPU with 16GB RAM.

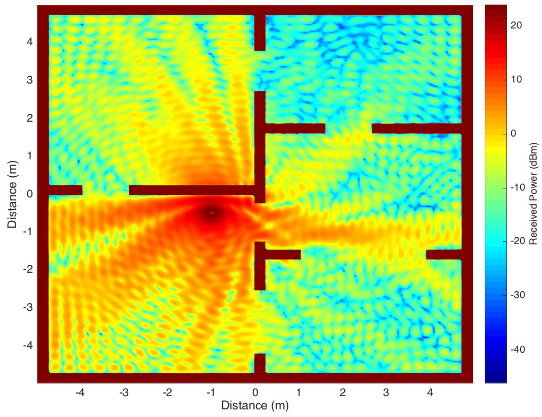

Fig. 1. Received power predicted throughout building by the 3D VEFIE. The building is of size 10m $\times$ 10m $\times$ 3m with concrete walls characterised by a relative permittivity of $\epsilon_r = 4.4$ and a conductivity of $\sigma = 0.01$. A dipole radiating at 700MHz is positioned at $(-1, -0.5, 0)$.

TABLE I
ACCURACY AND RUNTIME OF PROPAGATION MODELS.

Model	Runtime (mins)	RMSE (dB)	Std. Dev. (dB)
3D VEFIE	181		
Heuristic	5.7	4.25	4.10
Hybrid	1.2	5.28	4.93
2D VEFIE	1.1	27.12	4.86
3D Ray Tracing	4.4	6.41	6.07

The received power predicted by the 3D VEFIE is shown in Figure 1. The building is of size 10m $\times$ 10m $\times$ 3m with concrete walls characterised by a relative permittivity of $\epsilon_r = 4.4$ and a conductivity of $\sigma = 0.01$. A dipole radiating at 700MHz is positioned at $(-1, -0.5, 0)$. Table I shows the time taken to compute the received power for the 2D and 3D VEFIEs, the 2D to 3D models and ray tracing for the plane $z = 0$. It can be clearly seen from Table I that both the 2D to 3D models produce predictions in a significantly quicker runtime than the full 3D VEFIE. They are of the same order as the 3D ray tracing model, which only solves for the plane $z = 0$ as well. This is principally due to their 2D nature.

Table I also provides a comparison of the accuracy provided by the 2D to 3D models against the VEFIE and ray tracing. The 3D VEFIE is the model being compared against because it solves Maxwell's equations exactly except for numerical inaccuracies and has been validated against the Mie Series, an exact analytic solution, and measurements. We can see in Table I, that the 2D VEFIE produces a large RMS error. The effects of this can be seen in Figure 2 where the average received power level of the 2D VEFIE does not match that of the 3D model. The 2D to 3D models are designed to correct for this which can be seen in Table I where they produce RMS error values around 4 - 5 dB as opposed to 27.12 dB for the 2D VEFIE. This is also evident in Figure 3 which compares the 2D to 3D models against the 2D and 3D VEFIE. It should also

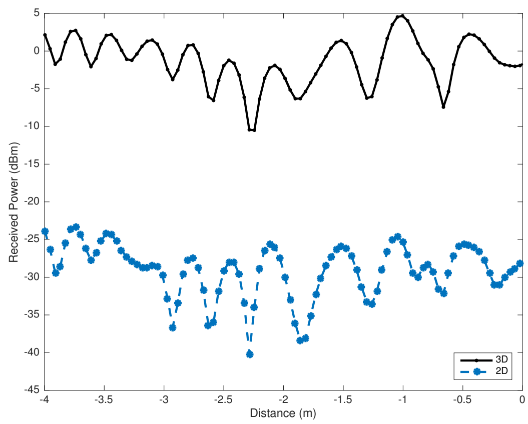

Fig. 2. Comparison of predicted received power by the 2D and 3D VEFIE models along the line $y = -3$ in the building shown in Figure 1.

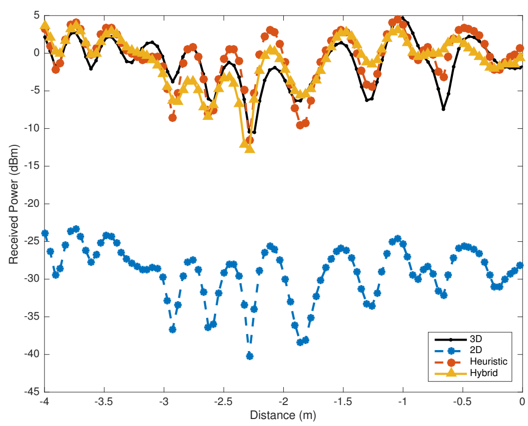

Fig. 3. Comparison of the predicted received power by the 2D to 3D models against the 2D and 3D VEFIEs along the line $y = -3$ in Figure 1.

be noted both the 2D to 3D models produce smaller RMS error and standard deviation values than 3D ray tracing. The hybrid model is also faster than the ray tracing model too. The 2D to 3D models are compared against the 3D ray tracing model in Figure 4 where it can be clearly seen they provide a higher level of accuracy and match the 3D VEFIE more closely.

V. Conclusions

In this paper an accurate full wave propagation model is presented for indoor environments. The model is based on the volume electric field integral equation (VEFIE). It is shown that the 3D version of the model is very slow whereas the 2D variant is quick but lacks accuracy. Two 2D to 3D models are presented to correct the accuracy of the 2D VEFIE and produce rapid accurate 3D predictions. The models are compared against the 3D VEFIE and shown to provide a high level of accuracy. They are significantly quicker than the 3D VEFIE and run in a similar time frame to ray tracing, the current most popular indoor propagation model. The 2D to

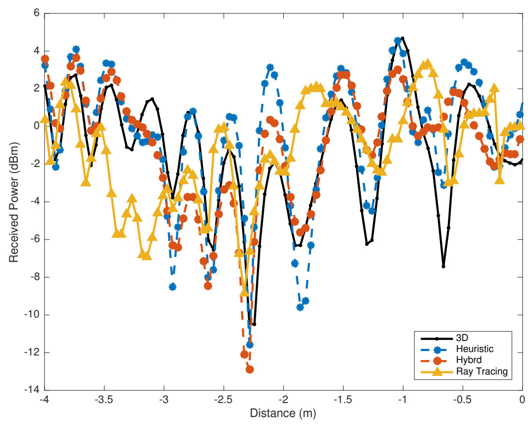

Fig. 4. Comparison of the predicted received power by the 2D to 3D models and ray tracing against the 3D VEFIE along the line $y = -3$ in Figure 1.

3D models are also more accurate. In the future we hope to develop a more rigorous and accurate 2D to 3D model as well as validating them against measurements.

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