A full wave propagation model for indoor wireless communications

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***Abstract*—This paper presents a 2D full-wave solver for the problem of electromagnetic wave propagation in indoor environments. The solver is based on the Volume Electric Field Integral Equation (VEFIE) formulation which when discretised by the method of moments results in a linear system whose iterative solution can be accelerated by using the FFT. In addition we explain how a simple pre-multiplication can force the iterative solver to focus on computing the values of fields in the scatterers, reducing the number of iterations required and leaving the computation of fields in free-space as a simple post-processing. Numerical results are presented validating the model against the Mie series and Uniform Theory of Diffraction. Some sample building simulations are presented.**

I. Introduction

The ability to accurately describe electromagnetic wave propagation underpins all radio channel modelling and wireless system development. The indoor environment presents its own unique challenges given the strong potential for multipath and variety of materials encountered. New developments in energy-efficient wireless communications such as the deployment of femto-cells and location and tracking algorithms that incorporate multipath information have created a greater demand for accurate propagation models that can include as much of the physics of the environment as is possible but run in reasonable compute times. Models to date have tended to be empirical, or if deterministic, based on ray tracing. There has been some efforts to develop full-wave models based on for example Finite Difference Time Domain algorithms. This paper outlines recent work which creates a very accurate model by applying the 2D volume integral equation formulation. This is a frequency domain formulation which when discretised produces a matrix equation which can be iteratively solved for the electric field on a regular lattice of N points throughout the environment. The form of the kernel of the integral operator means that the Fast Fourier Transform can be used to reduce the cost of each iteration to $\mathcal{O}(N\log N)$ from $\mathcal{O}\left(N^2\right)$. In addition a simple pre-multiplication step allows the algorithm to concentrate solely on solving for the unknowns located in the scatterers (walls etc) ignoring the unknown field values in free-space which can be computed in a final post-processing step. This has the effect of dramatically reducing the number of Conjugate Gradient iterations needed to solve the matrix equation. An alternative approach is to use a more sophisticated solver such as the BiCGSTAB. This document is organised as follows: Section (II) introduces the Volume Electric Field Integral Equation along with its discretisation using the method of moments and its compatibility with the Fast Fourier Transform. Section (III) presents validation results whereby the full-wave results are compared to those obtained from the Mie series. In addition we compare the predicted fields against those predicted by the Uniform Theory of Diffraction (UTD) for simple cases where we expect UTD to be accurate. We then show how the full-wave model can be applied to more complicated scenarios where UTD fails. Section (IV) examines the effect of the choice of iterative solver. Section (V) discusses some large-scale simulation results. Section (VI) offers conclusions and identifies future work needed.

II. Formulation

An antenna (modelled as a line source in this 2D implementation) is placed at some (x,y) location in a building and radiates at a fixed frequency. Waves (with the electric field polarised in the z direction) emanate from the antenna and propagate through, and scatter from, the various walls, doors, windows and apertures. Our challenge is to find the fields throughout the environment. The Volume Electric Field Integral Equation (V-EFIE) [1] writes an equation for the volume currents in the scatterers in terms of potentials. The V-EFIE can be expressed in slightly different form from that given in the reference above, namely

$$E_z^{inc}(\vec{r}) = E_z(\vec{r}) + \frac{\jmath}{4}\int O(\vec{r}\,')\,E_z(\vec{r}\,')\,H_0^{(2)}\left(k_0\left|\vec{r}-\vec{r}\,'\right|\right)ds' \quad (1)$$

where the contrast function is given by

$$O(\vec{r}) \equiv k^2(\vec{r}) - k_0^2 \quad (2)$$

and measures the difference between the wavenumber at a point and the wavenumber in free space. We apply the method of moments by introducing N pulse basis functions, defined on a regular (x,y) grid

$$E_z(\vec{r}) = \sum_{n=1}^{N} e_n p_n(\vec{r}) \quad (3)$$

where

$$p_n(r) = \begin{cases} 1 & \vec{r} \in \Delta_n \\ 0 & \vec{r} \notin \Delta_n \end{cases} \quad (4)$$

and Δ_n denotes the domain of pulse basis function n. Point matching at the centre of each cell yields the linear system

$$\mathbf{Ae} = \mathbf{v} \tag{5}$$

where $\mathbf{A}$ is a $N \times N$ impedance matrix, $\mathbf{v}$ is a $N \times 1$ vector containing incident field information and $\mathbf{e}$ is a $N \times 1$ vector containing the unknown electric fields. Equation (5) can be solved by direct inversion for small problems. However it is more common to use iterative methods based on Krylov sub-spaces for medium to large problems. In this work we use the Conjugate Gradient with Normal Equations (CG-NE) method and BiCGSTAB. Iterative schemes require repeated pre-multiplication by $\mathbf{A}$ of a sequence of trial solutions $\mathbf{e}_0, \mathbf{e}_1, \ldots$ until some convergence criterion is met. The particular form of $\mathbf{A}$ makes this quite computationally efficient as

$$\mathbf{A} = \mathbf{I} + \mathbf{GD} \tag{6}$$

where $\mathbf{I}$ is a $N \times N$ identity matrix, $\mathbf{D}$ is a $N \times N$ diagonal matrix while $\mathbf{G}$ is block Toeplitz which means we can use the Fast Fourier Transform (FFT) to pre-multiply a vector by it. Hence the computational cost of multiplying a vector by $\mathbf{A}$ reduces to $\mathcal{O}(N \log N)$ instead of $\mathcal{O}\left(N^2\right)$. This represents quite a saving when N is large.

III. Validation

The code was validated by comparing it against the Mie series solution for scattering from a dielectric cylinder (2D sphere). The incident field was a plane wave propagating in the x direction with frequency 700MHz. The cylinder had radius $0.25m$, $\epsilon_r = 10$ and was centred at $(0, 0)$. Figure (1) shows the total electric field along a vertical line through the cylinder, obtained using both the Mie series and the V-EFIE. A discretisation rate of 20 samples per wavelength was used in order to adequately capture the curvature of the cylinder. In practice a less dense sampling rate of 10 samples per wavelength is used for simulating propagation in a building. Excellent agreement is obtained between the two results. The second validation example is depicted in Figure (2). A line source illuminates a strongly reflecting cube with $\epsilon_r = 5$ and $\sigma = 0.5$. The VEFIE with 10 basis functions per wavelength is used to compute the fields along the black line. These results are compared to those obtained using the Uniform Theory of Diffraction (UTD). Note the existence of three distinct regions along the black line. The lower region consists of points where only a diffracted field (emanating from the north east corner of the scatterer) exists. Above that is a region where there are LOS incident rays as well as the diffracted region, while at the top of the line is a region where there are direct, reflected and diffracted fields. Figure (3) shows the excellent agreement between the two predictions (the fields are effectively superimposed in the diagram).

Our final validation compares the VEFIE fields to those produced by ray theory when applied to propagation through a slab. Consider the geometry shown in figure (4). A lossless slab of width $0.25m$ and length $2m$ (from $-1 < y < 1$) has

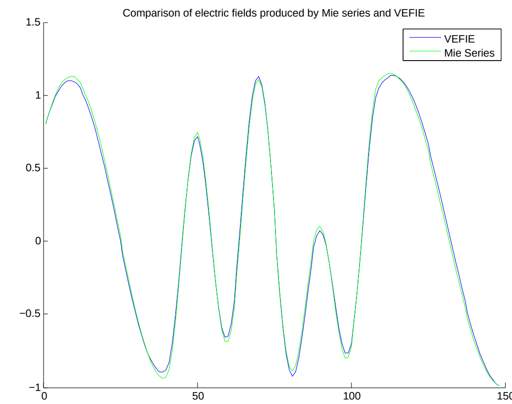

Fig. 1. Comparison of total fields along vertical cut through dielectric cylinder with $\epsilon_r = 10$, showing agreement between V-EFIE solution and that of Mie series

permittivity $\epsilon_r = 3.5$. A tapered plane wave propagates in the x direction with electric field given by

$$E_z(x, y) = \left(1 - y^2\right) e^{-\jmath k_0 x} \tag{7}$$

The tapering ensures that the incident field is zero at the top and bottom of the slab, minimising (but not eliminating) diffraction from the edges and ensuring that propagation through the slab along with reflections from the front face, as well as internal reflections, are the dominant propagation mechanisms. Figure (5) compares the fields predicted by the VEFIE (red) to those obtained by considering a single reflection from the front face of the slab (blue) as well as those obtained by considering all multiple order reflections within the slab (black). Good agreement is achieved between the VEFIE and the more realistic physical model. The validation examples have demonstrated that the full wave solver has been correctly implemented. However in all cases presented the reference solutions (Mie series, ray theory) are adequate, and computationally less complex. In order to motivate where the VEFIE may be useful consider the geometry shown in figure (6). Here cavities have been inserted into the slab. The multiple internal reflection model does not account for these, while the VEFIE can naturally incorporate them (by adjusting the contrast to zero in the relevant cells). There is no longer good agreement between the predictions, as the ray model cannot account for the effect of the internal structure within the wall.

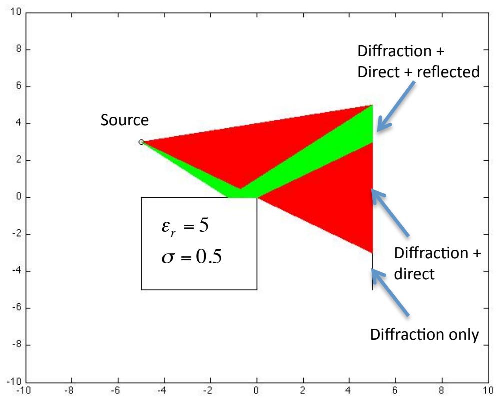

Fig. 2. Geometry of comparison with UTD. Receiver points are along black line. Geometric optics rays shown to distinguish 3 separate optical regions. Red lines show line of sight (LOS) rays, green lines are reflected rays. Diffracted rays (not shown) emanate from the north east corner of the scatterer to all points on the black line.

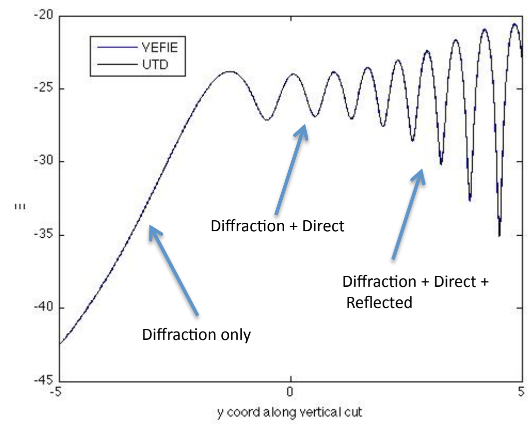

Fig. 3. Electric field (in dB) along line of receivers. From the left we see a smooth region where there is only a diffracted field, leading to an interference region where there is diffracted plus direct and in the final interference region diffracted, direct and reflected.

IV. Reduced forward operator

In the VEFIE formulation it is necessary to discretise the entire region of interest including free space. This is in contrast to boundary element integral equation formulations that only discretise the surface of the scattering objects (assuming they are homogeneous). At first glance the VEFIE thus appears to greatly increase the number of unknowns needed to describe the problem. However one should note that discretisation of the entire region facilitates the use of the FFT to expedite matrix vector multiplications. Secondly while the number of unknowns is increased, unknowns located in free-space do not affect the value of unknowns anywhere else in the grid (that is they do not produce scattered field as they have zero contrast) and can be effectively removed from the CG optimisation process. This is done by pre-multiplying equation (5) by a diagonal matrix $\tilde{I}$ whose diagonal entries are given by

$$\tilde{I}_{mm} = \begin{cases} 1 & \forall m \text{ such that } O\left(\vec{r}_m\right) \neq 0 \\ 0 & \forall m \text{ such that } O\left(\vec{r}_m\right) = 0 \end{cases} \tag{8}$$

Note that this pre-multiplication does not compromise the ability to use the FFT for rapid matrix-vector multiplication as it merely introduces a trivial extra multiplication by a diagonal matrix at each iteration. The net effect, when applied to the CGNE, is a reduction in the number of iterations needed to solve the problem as compared to the un-reduced system. Interestingly, applying this reduction technique to the BiCGSTAB solver has no effect - the more sophisticated BiCGSTAB solver already ignores unknowns that are not

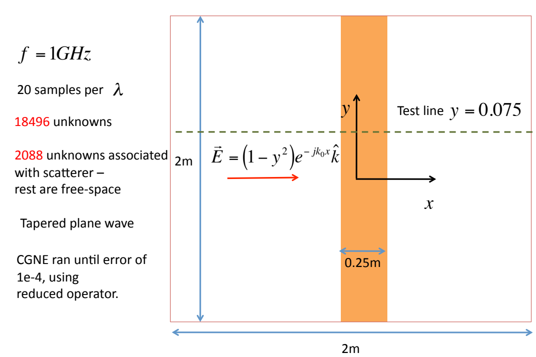

Fig. 4. Geometry for propagation through slab.

influencing others and converges more quickly than the accelerated CGNE. Example simulations will be discussed during the presentation.

V. Application to indoor environment

The model has been applied to realistic indoor environments. Figure (8) shows results obtained when the model is applied to a $10m \times 10m$ indoor space centred on $(0, 0)$ and comprising 5 rooms with lossy concrete walls. 10 samples per wavelength were used and the frequency was 1GHz leading to a total of 678,976 unknowns, of which 578,986 were free-space unknowns. The source was located at $(-0.8, -0.8)$. A tolerance of 10^{-3} was used for the CG-NE and the simulation converged after just over 1000 iterations, running on a 2008 Mac Book Pro with 4GB of RAM and a 2.53GHz processor.

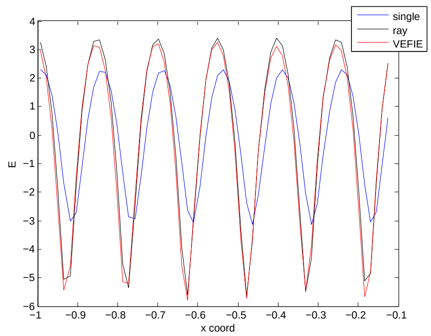

Fig. 5. Fields along test line to the left of the slab. VEFIE matches multiple reflection model, while single reflection model not so accurate.

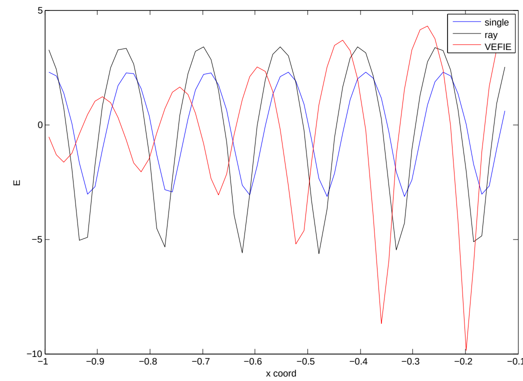

Fig. 7. Fields along test line to the left of the slab with cavities. Simple multiple reflection model no longer able to model internal structure and field predictions no longer match.

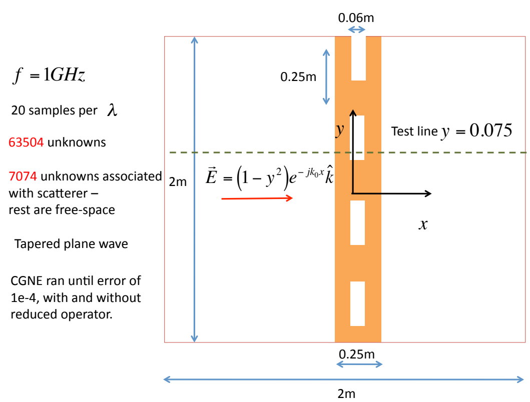

Fig. 6. Slab with internal cavities.

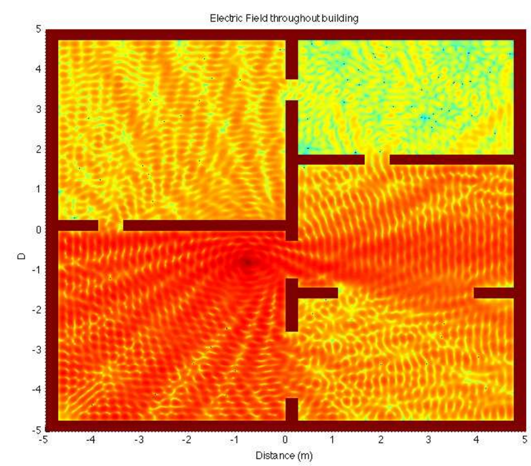

Fig. 8. Electric fields throughout 2D building environment.

VI. Conclusions and Future Work

A description has been given of initial attempts to develop a full-wave solver for indoor EM wave propagation. The model is based on the Volume Electric Field Integral Equation which allows the use of the FFT to speed up iterations. The increase in the number of unknowns does not have any adverse effect on the number of iterations required to solve the system. The model has been validated against the Mie series and UTD solutions. A motivating example showing propagation in the vicinity of a wall with cavities shows how the VEFIE can readily model the internal structure, something not easily done with ray methods. Future work will see the model being applied to 3D problems, and a wideband version developed possibly using AWE [3] to develop a rapid accurate approximation to the fields over the frequency band of interest.

References

[1] R. Mittra, A. Peterson and S. Ray,"Computational Methods for Electromagnetics", IEEE Press Series on Electromagnetic Wave Theory, 1998.

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[3] C. Brennan, P. Bradley and M. Condon."Efficient wideband electromagnetic scattering computation using well-conditioned asymptotic waveform evaluation", IEEE Transactions Antennas and Propagation, Vol. 57, No. 10, pp.3274-3282. October 2009.